**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: The work begins after 10 min, so average of work time increases 45min to 55min

For ND, 1 hour=60min

Z=(X-mu)/sigma

= (60-550)/8

= 0.625

By using z table, we get=0.7324

He asks the probability that the service manager cannot meet his commitment= 1-0.7324

= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS: Mean=38

SD=6

Z score=(X-mu)/sigma

=(44-38)/6

=6/6

=1

By using z table=84.13%

People older than 44 age=100-84.13

=15.87%

=63/400

For 38= (38-38)/6 = 0

By using z table = 50%

People between 38 and 44=84.13-50

=34.13%

=137/400

Therefore, this statement is **FALSE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: z score for 30= (30-38)/6

= -1.33

= 9.15%

=36/400

Therefore, this statement is **TRUE.**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: for 2X1 distribution and parameters will be N(2mu,2^2sigma^2)

For X1+X1 distribution and parameters will be N(2 mu, 2 sigma^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANS: 100-99=1%

0.5%

Z score for (0.005) =-2.576

Z value for 99.5% = 2.576

Z=(X-100)/20

X=20z+100

a= (20\*-2.576) +100=48.5

b= (20\*2.576) +100=151.5

correct option is D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: total profit = profit1 +profit2=P

Hence p=N (12,74) =p-12/sqrt (74)N(0,1)

P-12/Sqrt (74) =0.95

=1.644

P=12+1.644\*sqrt (74)

=13.41 (standard normal distribution)

$13.41=45\*13.41

=rs.603.68

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: pr(p<q)=5%

PR (p-12/sqrt (74) <q-12/sqrt (74)) =0.05

From standard normal distribution table

p-12/sqrt (74) =-1.644

p=12-1.644\*sqrt (74)

= 10.585

45\*10.585=rs.476.33

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: for 1st division

pr (profit1-5/sqrt (32) <0-5/sqrt (74))

= 0.1883

For second division

PR (profit2-7/sqrt (42) <-1.080)

=0.1400

The division that has a larger probability of making a loss in given year is the first division.